

4-2 Operations with Matrices
4-3 Multiplying Matrices



Norcom is a company that runs 3 factories to produce their 4 most popular products.

During the first week of the month, their output, measured in units, is as follows:

	Product 1	Product 2	Product 3	Product 4
Factory 1	6	3	2	0
Factory 2	0	4	8	5
Factory 3	4	2	1	0

Put the following data into a matrix.

$$A = \begin{bmatrix} 6 & 3 & 2 & 0 \\ 0 & 4 & 8 & 5 \\ 4 & 2 & 1 & 0 \end{bmatrix}$$

During the second week of the month, their production schedule changes and their output, measured in units, is as follows:

	Product 1	Product 2	Product 3	Product 4
Factory 1	3	3	0	1
Factory 2	0	6	5	2
Factory 3	10	2	0	1

Put the following data into a matrix.

$$B = \begin{bmatrix} 3 & 3 & 0 & 1 \\ 0 & 6 & 5 & 2 \\ 10 & 2 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & 3 & 2 & 0 \\ 0 & 4 & 8 & 5 \\ 4 & 2 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 3 & 0 & 1 \\ 0 & 6 & 5 & 2 \\ 10 & 2 & 0 & 1 \end{bmatrix}$$

What is the total production for each product at each factory after 2 weeks? (express in a matrix)

$$A+B = \begin{bmatrix} 9 & 6 & 2 & 1 \\ 0 & 10 & 13 & 7 \\ 14 & 4 & 1 & 1 \end{bmatrix}$$

Matrix addition and subtraction

- matrices must have the same dimensions
- each element is added/subtracted to the element in its corresponding location

$$A = \begin{bmatrix} 6 & 3 & 2 & 0 \\ 0 & 4 & 8 & 5 \\ 4 & 2 & 1 & 0 \end{bmatrix} \quad 2 \times B = \begin{bmatrix} 3 & 3 & 0 & 1 \\ 0 & 6 & 5 & 2 \\ 10 & 2 & 0 & 1 \end{bmatrix}$$

Suppose Norcom needs to meet a large order, how much production would they have if, during the second week, they are open twice as long? (express in a matrix)

$$A + 2B = \begin{bmatrix} 6 & 6 & 0 & 2 \\ 0 & 12 & 10 & 4 \\ 20 & 4 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 9 & 2 & 2 \\ 0 & 16 & 18 & 9 \\ 24 & 6 & 1 & 2 \end{bmatrix}$$

Scalar Multiplication

- multiplying every element by the same value

Matrix Multiplication

- If $A \times B$, then # columns for A = # rows for B
- If $A_{m \times n} \times B_{n \times p}$, then $AB_{m \times p}$
- Every element in a row from A is multiplied by every element in a column from B and then added together

$$A_{2 \times 3} \times B_{3 \times 4} = AB_{2 \times 4}$$

$$C_{2 \times 6} \times D_{6 \times 1} = CD_{2 \times 1}$$

$$E_{8 \times 3} \times F_{8 \times 4} \quad \text{Not possible}$$

Example:

$$C = \begin{bmatrix} 5 & 4 & 8 \\ -2 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} -2 & 0 \\ 6 & 4 \\ 1 & -3 \end{bmatrix}$$

CD 2×2 3×2 2×3 3×3

$$D \times C = DC$$

$$= \begin{bmatrix} 5(-2) + 4(6) + 8(1) & 5(0) + 4(4) + 8(-3) \\ -2(-2) + 0(6) + 1(1) & -2(0) + 0(4) + 1(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -8 \\ 5 & -3 \end{bmatrix}$$

$CD = \begin{bmatrix} 22 & -8 \\ 5 & -3 \end{bmatrix}$

ex:

$$E = \begin{bmatrix} 7 \\ 3 \end{bmatrix} \quad F = \begin{bmatrix} 1 & 5 \\ -3 & 2 \\ 0 & 1 \\ 4 & -2 \end{bmatrix} \quad E = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$E \cdot F$ = not possible

$F \times E$ 4×1

$$\begin{bmatrix} 1(7) + 5(3) \\ 3(7) + 2(3) \\ 0(7) + 1(3) \\ 4(7) + (-2)(3) \end{bmatrix} = \begin{bmatrix} 22 \\ -15 \\ 3 \\ 22 \end{bmatrix}$$

$\begin{bmatrix} 1/1 \\ 2/1 \\ 3/1 \\ 4/1 \end{bmatrix}$

Suppose the profit for each unit of products 1-4 is: 3, 10, 7, and 2 respectively.

What would be the total profit for each factory during the first week?

$$A = \begin{bmatrix} 6 & 3 & 2 & 0 \\ 0 & 4 & 8 & 5 \\ 4 & 2 & 1 & 0 \end{bmatrix} \quad P = \begin{bmatrix} 3 \\ 10 \\ 7 \\ 2 \end{bmatrix}$$

3×4 4×1

$$AP = \begin{bmatrix} 6(3) + 3(10) + 2(7) + 0(2) \\ 0(3) + 4(10) + 8(7) + 5(2) \\ 4(3) + 2(10) + 1(7) + 0(2) \end{bmatrix}$$

$$AP = \begin{bmatrix} 62 \\ 106 \\ 39 \end{bmatrix}$$

Suppose the volume for each unit of products 1-4 is: 8, 5, 1, and 4 respectively.

What would be the total storage space used for each factory during the first week?

$$A = \begin{bmatrix} 6 & 3 & 2 & 0 \\ 0 & 4 & 8 & 5 \\ 4 & 2 & 1 & 0 \end{bmatrix} \quad V = \begin{bmatrix} 8 \\ 5 \\ 1 \\ 4 \end{bmatrix}$$

3×4 4×1

$$AV = \begin{bmatrix} 6(8) + 3(5) + 2(1) + 0(4) \\ 0(8) + 4(5) + 8(1) + 5(4) \\ 4(8) + 2(5) + 1(1) + 0(4) \end{bmatrix} = \begin{bmatrix} 65 \\ 48 \\ 93 \end{bmatrix}$$

If the the manager wanted to combine those calculations, she can as follows:

Calculator

$$A = \begin{bmatrix} 6 & 3 & 2 & 0 \\ 0 & 4 & 8 & 5 \\ 4 & 2 & 1 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 3 & 8 \\ 10 & 5 \\ 7 & 1 \\ 2 & 4 \end{bmatrix}$$

3×4 4×2

Homework:

p.164

14, 15, 25, 27

p172-173

13-20(hand), 31-34, 36- 39